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REMARKS ON NUCLEATE BOILING HEAT TRANSFER FROM A HORIZONTAL SURFACE

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Abstract—In the first part of the paper a hydrodynamical model is proposed for the regime of isolated bubbles. The model which is of the boundary-layer type takes into account the circulation motion caused in the liquid by the bubbles which break off. Two equations are obtained, one holding for laminar, the other for turbulent flow.

In the second part a mechanism of the heat transfer at large heat fluxes is proposed implying the existence of vapour columns and the motion of a thin liquid film under these columns. In this mechanism it is considered that the rate of heat transfer is dependent on: (1) the hydrodynamic interaction between the liquid threads moving towards the heating surface and the vapour columns; (2) the motion of the liquid as a thin layer under the vapour columns, under the action of surface forces; (3) the vaporization of this film which takes place at the liquid–vapour interface and explosively in the neighbourhood of the active centres.

NOMENCLATURE

- a, thermal diffusivity;
- c, specific heat of the liquid;
- D_o $= 2R_o$;
- g, gravitational acceleration;
- h, h', heat-transfer coefficient;
- $\bar{h}$, heat-transfer coefficient corresponding to the area under a vapour column;
- h_1, heat-transfer coefficient corresponding to the area under the bulk of the liquid;
- J, mechanical equivalent of heat;
- k, thermal conductivity;
- l_o, average value of the sizes of the surface cavities;
- L, average distance between successive active centres;
- L_o, average distance between successive cavities;
- n, number of active centres per unit area;
- p, pressure;
- q, heat-flux density;
- q_{max}, maximum heat-flux density;
- Q, volumetric flow rate per unit breadth;
- Q_o, value of Q for $x = 0$;
- r, latent heat of vaporization;
- R_o, radius of the bubble at the moment when the bubble breaks off;
- R_o', radius of vapours columns;
- T_l, temperature in the bulk of the liquid;
- T_w, temperature at the wall;
- T, T^*, temperatures on the two sides of the liquid film–vapour interface;
- ΔT, temperature difference between the wall and the bulk of the liquid;
- u, component of liquid velocity along the x–axis;
- u_o, liquid velocity at the beginning of the interval of length $[(4/\pi)^{1/2}L - R_o]/2$;
- u_1, liquid velocity at the beginning of a path of length x_1;
- U, terminal bubble rise velocity;
- x, distance along the plate measured from the boundary between a vapour column and the liquid;
- x_o, value of x for which $\delta = 0$;
- x_1, path length of an element of liquid along the boundary in turbulent conditions;
- y, distance from the wall;

Greek symbols
- α, stagnation flow constant;
- β, vaporization or condensation coefficient;
- δ, film thickness;
- δ_o, value of δ at the boundary between the liquid and a vapour column;

ϵ, volumetric vapour fraction;
ϵ', surface fraction occupied by vapour columns;
ρ', liquid density;
ρ'', vapour density;
σ, dynamic surface tension;
σ_{lv}, liquid–vapour surface tension;
σ_{ls}, liquid–solid surface tension;
σ_{sv}, solid–vapour surface tension;
χ $\equiv (\sigma_{sv} - \sigma_{ls}) - \sigma_{lv} \cos \theta'$;
η, dynamic viscosity;
ν, kinematic viscosity;
θ, contact angle between a bubble and the solid surface (in degrees);
θ', angle between the free surface of the liquid film and the wall at $x = x_0$.

RECENT experimental results have shown that there are two regimes of nucleate boiling: (a) the regime of isolated bubbles and (b) the interference regime [1, 2]. The dominant heat-transfer mechanism differs essentially in the two regimes; in the first it is due to the stirring action of the bubbles, while in the second it is due to the latent heat transport.

For the first regime theoretical equations have been established which take into account both the hydrodynamic process [3–8] and the nucleation properties of the heating surface [3]. The hydrodynamic process has been represented either by considering the movement caused within the liquid by the bubbles which grow on the active centres of the heating surface [4–7], or by considering the circulation motion caused by the bubbles which break off [2, 8].

Concerning the latter representation, two points of view have been developed. Tien [8] proposes a hydrodynamic model of stagnation laminar flow; the assumption that the characteristic constant of stagnation flow depends on n and ν leads him, via dimensional considerations, to an equation for this constant and therefore for the heat-transfer coefficient. Zuber [2] has obtained an equation for the heat-transfer coefficient on the basis of an analogy with turbulent free convection, replacing the buoyant force in the equation for the free convection by the difference between the liquid specific weight and the specific weight of the two phase mixture. Though the starting point of the two theories is the same, the results differ, the one obtained by Zuber being in better agreement with experiment. The attempt to explain this disagreement has led us to a point of view which may be considered as a synthesis of the two aforementioned theoretical approaches; as in Tien's work, hydrodynamical models are proposed, but the hydrodynamical parameters appearing in the equations written on the basis of these models are expressed as functions of quantities, more significant for the process, suggested by Zuber's analogy. Two hydrodynamical models are used: one valid for laminar motion which is similar to that of Tien, and the other one for turbulent motion (a case not considered by Tien). The equation obtained for the turbulent flow coincides with the relation established by Zuber (2) on the basis of the above mentioned analogy.

For the second regime, Moore and Mesler [9] have suggested a mechanism which implies the existence of a static thin liquid film on the heating surfaces under the bubbles. The evaporation of this film determines the rate of heat transfer. In the present paper, arguments are put forward in favour of another mechanism implying the existence of vapour columns instead of growing bubbles and the motion of a thin liquid film under these columns, the motion being under the action of surface forces.

REGIME OF ISOLATED BUBBLES

(1) Jakob's experiments [10, 11] have shown that, in the neighbourhood of a growing and departing bubble, strong forward and backward movements of the liquid take place. As the number of active centres and the frequency of bubble generation increases, the bubble columns entrain the liquid in a nearly continuous way. This ascending movement of the liquid is compensated by a descending movement in the central region of the space between the bubble columns and is continued in the neighbourhood of the heating surface with a movement quasi-parallel to the latter. It has been suggested [3, 6] that around every active centre a domain having a square form may be ascribed, the area of which $\approx L^2$, L being the distance between two successive active centres. The previously mentioned experiments have shown that the descending liquid reaches the heating surface in the vicinity of the

boundary of the domain of area $\approx L^2$; from there on the movement becomes quasi-horizontal and then ascending in the vicinity of the bubbles which break off from the active centre. The rate of heat transfer is determined by the hydrodynamic process which takes place in the vicinity of the heating surface.

(2) Let us assume at first that the motion along the plate is laminar. The distance covered by an element of liquid along the plate cannot be calculated exactly; we shall take as its mean value the difference $\frac{1}{2}[(4/\pi)^{1/2}L - R_o]$. Though the motion of the liquid is quasi-radial, we will assume, for simplification, that it may be considered as the motion of a liquid along a plate. The distance $\frac{1}{2}[(4/\pi)^{1/2}L - R_o]$ being in general small, it may be probably considered that the thicknesses of the hydrodynamic and thermal boundary layers (computed by means of equations holding for a semi-infinite fluid) is smaller than the thickness of the moving liquid elements. Therefore the equations holding for a semi-infinite liquid can be applied. Thus we obtain for the heat-transfer coefficient, defined as an average value over the interval $\frac{1}{2}[(4/\pi)^{1/2}L - R]$, the following expression:

$$h' \propto k \left(\frac{u_o}{\nu\,[(4/\pi)^{1/2}L - R]}\right)^{1/2} \left(\frac{\nu}{a}\right)^{1/3}. \qquad (1)$$

Considering that the part of the heating surface which is covered by the bubbles has only a slight contribution to the heat transfer we can, in a first approximation, express the heat-transfer coefficient per unit surface area by the relation:

$$h \propto k \left[1 - \frac{\pi}{4}\left(\frac{R_o}{L}\right)^2\right] \times \left(\frac{u_o}{\nu\,[(4/\pi)^{1/2}\,L - R_o]}\right)^{1/2} \left(\frac{\nu}{a}\right)^{1/3}. \qquad (2)$$

The circulation motion described above is due to the buoyant force $g\Delta\rho$, where $\Delta\rho$ is the difference between the density of the liquid and the density of the two phase mixture in the vicinity of the wall. It depends also on the physical constants of the liquid. The velocity u_o depends on the velocity of the circulation motion and therefore on the buoyant force

$$\rho' g - [\rho'(1-\epsilon) + \rho''\epsilon]g \equiv (\rho' - \rho'')g\epsilon$$

and on the physical constants of the liquid (kinematic viscosity ν, density ρ'). Since the number of physical constants is four while that of the dimensions involved is three, it follows that one may form a single dimensionless group. A simple calculation leads to:

$$u_o \propto \left(\frac{\rho' - \rho''}{\rho'}\, g\epsilon\nu\right)^{1/3}. \qquad (3)$$

It may be mentioned that the velocity u_o increases both with the buoyant force and with the viscosity. The increase with viscosity is due to the fact that for a given buoyant force the quantity of liquid entrained by the bubble columns is larger for large values of the liquid viscosity.

Equation (2) becomes

$$h \propto k \left[1 - (\pi/4)\left(\frac{R_o}{L}\right)^2\right] \times \left(\frac{\rho' - \rho''}{\rho'}\,\epsilon\right)^{1/6} \frac{g^{1/6}}{a^{1/3}\,[(4/\pi)^{1/2}\,L - R_o]^{1/2}}. \qquad (4)$$

The volume fraction ϵ occupied by the bubbles in the vicinity of the heating surface may be expressed, under certain conditions, by the equation [2]

$$\epsilon \propto \frac{1}{L^2}\left(\frac{c\rho'\Delta T}{r\rho''}\right)^2 \frac{aR_o}{U} \qquad (5)$$

and therefore equation (4) takes the form:

$$h \propto k \left[1 - (\pi/4)\left(\frac{R_o}{L}\right)^2\right] \left(\frac{\rho' - \rho''}{\rho'}\right)^{1/6} \times \left(\frac{c\rho'\Delta T}{r\rho''}\right)^{1/3} \left(\frac{aR_o}{U}\right)^{1/6} \frac{g^{1/6}}{a^{1/3}\,[(4/\pi)^{1/2}\,L - R_o]^{1/2} L^{1/3}}. \qquad (6)$$

The velocity U may be computed by one of the equations proposed by Peebles and Garber [12] or Harmathy [13]. Under certain boiling conditions :

$$U = 1{\cdot}18 \left[\frac{\sigma g\,(\rho' - \rho'')}{\rho'^2}\right]^{1/4}. \qquad (7)$$

The diameter D_o of a bubble which breaks off is given by the equation [7]:

$$g(\rho' - \rho'')\,(\pi/6)\,D_o^3 = 4\pi \times 10^{-4}\,\theta^2\,D_o\sigma + (\pi/2)\,\rho' \times \left[\frac{c\rho'(\pi a)^{1/2}\Delta T}{r\rho''}\right]^4. \qquad (8)$$

The mean distance between two successive centres may be evaluated by the relation [3]

$$L = L_0 \psi^{-1/2} \tag{9}$$

where ψ is a function of the dimensionless group

$$\frac{\sigma_{lv} T}{l_0 r \rho'' J \Delta T}.$$

The form of the function ψ may be obtained if we know the distribution function of the cavities of the heating surface.

A complete comparison of equation (6) with the experimental results is not yet possible since the function ψ and the lengths L_0, l_0 are not known. It should be remarked however that if we neglect $(R_0/L)^2$ as compared with unity and consider that R_0 is not dependent on ΔT equation (6) becomes:

$$h \propto n^{5/12} (\Delta T)^{1/3} [1 \cdot 13 - R_0 n^{1/2}]^{-1/2} \tag{10}$$

which for not too large values of $R_0 n^{1/2}$ is in good agreement with the experimental results of Gaertner and Westwater [14] who have obtained $h \propto n^{0 \cdot 43}$.

The proposed model does not differ essentially from the one previously suggested by Tien and for this reason the latter will be discussed here.

Tien [8] has suggested a hydrodynamic model of stagnation flow and considering laminar flow has obtained:

$$h \propto k \left(\frac{\alpha}{\nu}\right)^{1/2} \left(\frac{\nu}{a}\right)^{1/3}, \tag{11}$$

where α is the characteristic constant of stagnation flow. Assuming further that α is dependent on n and ν, Tien shows on the basis of dimensional considerations that $\alpha \propto n\nu$ and therefore that*:

$$h \propto k n^{1/2} \left(\frac{\nu}{a}\right)^{1/3}. \tag{12}$$

* It seems more reasonable to assume that α depends on the quantities which characterize the circulation motion of the liquid, (i.e. the buoyant force $g(\rho' - \rho'')\epsilon$ and the physical constants of the liquid ν and ρ'), and therefore that $\alpha \propto [g\epsilon(\rho' - \rho'')/\rho']^{2/3}\nu^{-1/3}$. Equation (11) becomes:

$$h \propto k \left(\frac{g\epsilon(\rho' - \rho'')}{\rho' \nu^2}\right)^{1/3} \left(\frac{\nu}{a}\right)^{1/3} \tag{13}$$

Equation (13) is identical to that established by Zuber on the basis of the analogy between free turbulent convection and nucleate boiling.

It should be noted that owing to the similarity between stagnation flow and flat-plate boundary-layer flow, the model used here does not differ essentially from that used by Tien. However, the results obtained are different, especially because of the quantities chosen to characterize the state of motion of the liquid.

(3) We will now consider that the motion of the liquid is turbulent and analyse the heat transfer on the basis of the model suggested in reference [15] for the representation of a turbulent process in the vicinity of a solid boundary. In the model referred to, it is assumed that, owing to the turbulence, laminar boundary layers are formed in the immediate vicinity of the wall, over short successive paths of length x_1. In other words, the turbulent fluctuations cause the elements of liquid to come into contact with the wall; after travelling along it for short distances of length x_1, they "dissolve" in the bulk of the liquid. The process repeats itself at intervals of length x_1. For every such interval we apply the equation holding for a semi-infinite fluid in laminar motion along a plate. In this way one obtains the following equation for the heat-transfer coefficient defined as a mean value over the interval x_1:

$$h \propto k \left(\frac{u_1}{x_1 \nu}\right)^{1/2} \left(\frac{\nu}{a}\right)^{1/3}. \tag{14}$$

We remark that for the two cases (laminar and turbulent) the same equation is used. The difference consists in the fact that while in the laminar case the "laminar flow path" is determined by the bubble spacing, in the turbulent case the length x_1 of the "laminar flow path" is determined by the quantities which characterize the state of motion of the liquid.

The velocity u_1 and the path-length x_1 are dependent on the hydrodynamic state of the liquid in the vicinity of the heating surface, therefore on $g(\rho' - \rho'')\epsilon$ and the physical constants ν and ρ'. Dimensional considerations lead to

$$u_1 \propto \left(\frac{\rho' - \rho''}{\rho'} g \nu \epsilon\right)^{1/3}$$

$$x_1 \propto \left(\frac{\rho' - \rho''}{\rho'} g \epsilon\right)^{-1/3} \nu^{2/3}.$$

Equation (14) becomes

$$h \propto k \left(\frac{\rho' - \rho''}{\rho'} \frac{g \epsilon}{\nu^2}\right)^{1/3} \left(\frac{\nu}{a}\right)^{1/3}. \qquad (15)$$

Since the part of the heating surface which is covered by the bubbles has only a small contribution to the heat transfer, equation (14) has to be corrected. As a first approximation one may write

$$h \propto k \left[1 - (\pi/4)\left(\frac{R_0}{L}\right)^2\right] \left(\frac{\rho' - \rho''}{\rho'} \frac{g \epsilon}{\nu^2}\right)^{1/3} \left(\frac{\nu}{a}\right)^{1/3}. \qquad (15')$$

Equation (15) coincides with the equation established by Zuber [2] on the basis of the analogy between free turbulent convection and nucleate boiling. As shown by Zuber this equation is in good agreement with experiment.

INTERFERENCE REGIME

(1) If the heat flux becomes sufficiently large, a greater number of active centres contribute to the formation of a bubble, and for very large values of the flux (but smaller than the maximum flux) bubbles are no more distinguishable, but only vapour columns which seem to originate from the heating surface. By using a transparent heating surface and by taking photographs from below, Kirby and Westwater [16] have emphasized the existence, at high values of the heat flux, of large vapour patches connected to the surface by numerous vapour stems. Under each large vapour patch lies a very thin liquid layer in which dry spots appear and disappear.

What about the hydrodynamics of the process at high values of the flux? In the regime of isolated bubbles a circulation of the liquid goes on, the amount of liquid displaced towards the heating surface being generally large. At sufficiently high vapour flow rates the amount of liquid directed towards the heating surface decreases because the motion is delayed by the continuous jets of vapour (which, as the flux heat increases, cover an increasingly larger portion of the heating surface) and by the vapour velocity within these jets. At large heat fluxes the volumetric flow rate of liquid which is directed towards the heating surface is small [17–19].

There comes a critical moment when the number of jets and the vapour velocity become so great that only the amount of liquid which produces by vaporization the vapour flow is directed towards the heating surface.

This critical moment cannot be exceeded because if the vapour velocity were to surpass the critical velocity this would increase the braking action on the motion of the liquid towards the heating surface, and the liquid flow could no longer maintain the vapour flow.

Since at the maximum value of the flux the velocity of the liquid directed towards the heating surface may be neglected in comparison with that of the vapour, the hydrodynamic process which takes place may be considered similar to a flooding process. As a matter of fact the analogy between flooding and the maximum flux has been pointed out by Bonilla and Perry [20]. The moment of flooding may be determined quantitatively either by assuming that it takes place at the moment when the interfacial instability between vapour jets and liquid occurs, [19, 21] or by applying the flooding condition [22]. In both cases one obtains the equation

$$q_{\max} \propto r \rho'' \left[\frac{\sigma_{lv}\, g\, (\rho' - \rho'')}{\rho''^2}\right]^{1/4} \qquad (16)$$

which has already been found by Kutateladze [17] by means of dimensional considerations.

Equation (16) represents the only quantitative information concerning the interference regime.

(2) What is the heat-transfer mechanism at high fluxes? Moore and Mesler [9], when determining the wall temperature, have found that at high values of the heat flux, sudden temperature drops of 20°–30 degF take place in about 2 ms followed by a relatively slow temperature increase. The explanation put forward by these authors is that the growing bubbles do not remove all the liquid but leave at their bases a thin liquid film. The evaporation of this film causes the sudden drop of the wall temperature.

But growing bubbles should leave a liquid film on the heating surface also in the case of lower values of the heat flux, and therefore the phenomenon pointed out by Moore and Mesler should occur also for smaller fluxes; this has not been observed experimentally. A possible explanation which retains the main idea of Moore and

Mesler's work (viz. the existence of a liquid film) and includes also the experimental facts mentioned by Kirby and Westwater could be the following: There are no bubbles at high fluxes but only vapour columns. The liquid "threads" moving between the vapour columns towards the heating surface, on reaching the latter, continue their movement as a thin layer under the vapour columns. The motion of the thin layer of liquid takes place under the action of surface forces (surface wetting forces). In fact, in certain regions under the vapour columns the surface is uncovered by liquid. These "dry regions" determine the existence of surface wetting forces. The regions under the vapour columns contain active centres. The active centres probably make up some kind of "explosion centres" where nuclei are formed which favour the practically explosive evaporation of the adjoining liquid. This explosive evaporation absorbs heat from the solid wall thus bringing about a rapid temperature drop.

The rate of heat transfer is dependent in this case on: (1) the hydrodynamic interaction process between the vapour columns and the liquid threads; (2) the motion of the liquid film on the heating surface under the action of surface forces; (3) the vaporization process which takes place at the liquid–vapour interface of the film and possibly explosively in the neighbourhood of the active centres. (The nucleation process in the homogeneous or heterogeneous phase may also play a part if one considers the high temperatures involved.)

This representation of the process has some quantitative consequences. For the purpose of simplifying the computation we shall make the following approximations: (i) the part played by nucleation and active centres will be overlooked (this point will be discussed below); (ii) it will be assumed that the central part of the area under the vapour column is dry (although it has been shown experimentally [16] that the dry spots are distributed more or less uniformly under the vapour column); (iii) we shall write the equations for a plane motion and take no account of the quasi-radial symmetry. The model is sketched in Fig. 1.

As mentioned above the motion of the liquid film under the vapour columns takes place under

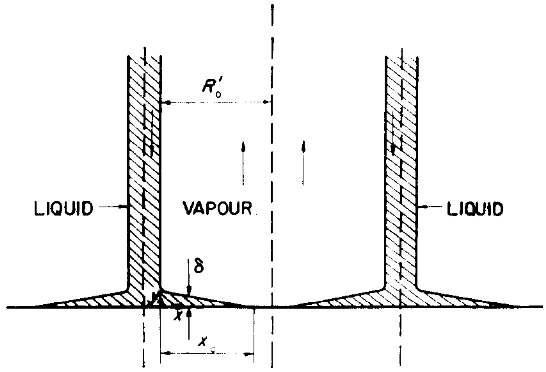

FIG. 1

the action of the surface wetting forces. The manner in which these forces act is not yet fully understood. It seems that these forces produce shear stresses at the liquid–vapour interface owing to a gradient along the interface of the dynamical surface tension. The motion of the liquid film is a consequence of these shear stresses. The wetting surface tension

$$\chi \equiv (\sigma_{sv} - \sigma_{sl}) - \sigma_{lv} \cos \theta'$$

is equal to the integral of shear stresses, taken over the whole liquid–vapour interface under a vapour column

$$\chi \approx \int_0^{x_0} \eta \left(\frac{\partial u}{\partial y}\right)_{y=\delta} \mathrm{d}x. \tag{17}$$

Neglecting the inertial forces, the equation of motion of the liquid film under the vapour column may be written

$$\eta \frac{\partial^2 u}{\partial y^2} - \frac{\partial p}{\partial x} = 0. \tag{18}$$

Continuity of normal stress at the interface permits us to write equation (18) in the form

$$\eta \frac{\partial^2 u}{\partial y^2} + \frac{\mathrm{d}}{\mathrm{d}x}\left(\sigma \frac{\mathrm{d}^2 \delta}{\mathrm{d}x^2}\right) = 0, \tag{18'}$$

where $\mathrm{d}^2\delta/\mathrm{d}x^2 \approx$ curvature of the interface.

The other boundary conditions are:

$$u = 0 \quad \text{for } y = 0 \tag{19}$$

and

$$\eta \left(\frac{\partial u}{\partial y}\right)_{y=\delta} = \frac{\mathrm{d}\sigma}{\mathrm{d}x} \text{ for } y = \delta. \tag{20}$$

In order to simplify the calculation we shall assume further that (i) the shear stress is uniformly distributed along the length x_o, and (ii) the term

$$\frac{d}{dx}\left(\sigma \frac{d^2\delta}{dx^2}\right)$$

in equation (18′) may neglected. The first simplifying assumption allows to obtain from equation (17)

$$\eta\left(\frac{\partial u}{\partial y}\right)_{y=\delta} \approx \frac{\chi}{x_o} \qquad (20')$$

and both simplifying assumptions lead to

$$u \approx \frac{\chi}{\eta x_o} y. \qquad (21)$$

The volumetric flow rate per unit breadth is given by

$$Q \equiv \int_0^\delta u\,dy \approx \frac{\chi\delta^2}{2\eta x_o}. \qquad (22)$$

The film thickness δ decreases as x increases, owing to the vaporization of the liquid under the effect of the heat flux. The rate of heat transfer is determined by the thermal conductivity through the liquid film and by the vaporization–condensation process which takes place at the liquid–vapour interface

$$q = \frac{k}{\delta}(T_w - T^*) = \beta(T^* - T) \equiv \frac{T_w - T}{\frac{\delta}{k} + \frac{1}{\beta}} \equiv \frac{\Delta T}{\frac{\delta}{k} + \frac{1}{\beta}}. \qquad (23)$$

Therefore

$$-r\rho'\frac{dQ}{dx} = \frac{\Delta T}{\frac{\delta}{k} + \frac{1}{\beta}}. \qquad (24)$$

Equations (22) and (24) lead to the differential equation

$$-\frac{r\rho'\chi}{\eta x_o}\delta\frac{d\delta}{dx} = \frac{\Delta T}{\frac{\delta}{k} + \frac{1}{\beta}}. \qquad (25)$$

Integrating, we get

$$\frac{r\rho'\chi}{\eta x_o}\left[\frac{\delta_o^3 - \delta^3}{3k} + \frac{\delta_o^2 - \delta^2}{2\beta}\right] = x\Delta T. \qquad (26)$$

Since $\delta = 0$ for $x = x_o$, equation (26) leads to

$$x_o = \left[\frac{r\rho'\chi}{\eta\Delta T}\left(\frac{\delta_o^3}{3k} + \frac{\delta_o^2}{2\beta}\right)\right]^{1/2} \qquad (27)$$

It may be stressed that from equations (22) and (27) result that Q_o is related to δ_o by

$$Q_o = \frac{\chi}{2\eta}\frac{\delta_o^2}{\left[\frac{r\rho'\chi}{\eta\Delta T}\left(\frac{\delta_o^3}{3k} + \frac{\delta_o^2}{2\beta}\right)\right]^{1/2}}. \qquad (28)$$

The mean heat-transfer coefficient for the area under the vapour column may be expressed by

$$\bar{h} = \left[1 - \left(1 - \frac{x_o}{R_o'}\right)^2\right]\left[\frac{1}{x_o}\int_0^{x_o}\frac{dx}{\frac{\delta}{k} + \frac{1}{\beta}}\right]. \qquad (29)$$

For $\beta \gg k/\delta$ we get

$$\bar{h} = \frac{3}{2}\left[1 - \left(1 - \frac{x_o}{R_o'}\right)^2\right]\frac{k}{\delta_o} \equiv \frac{g}{8}\frac{k^2\chi\Delta T}{Q_o^2\eta r\rho'}\left[1 - \left(1 - \frac{x_o}{R_o'}\right)^2\right]. \qquad (30)$$

The factor $(1 - (1 - [x_o/R_o'])^2)$ takes into account that part of the area under the vapour column which is occupied by the dry region. The mean heat-transfer coefficient over the whole area of the heating surface may be written as a sum:

$$h = \bar{h}\epsilon' + h_1(1 - \epsilon'). \qquad (31)$$

The heat-transfer coefficient is dependent both on the heat transfer through the thin liquid film which is moving under the action of surface forces, under the vapour columns, and on hydrodynamic factors related to the interaction between the vapour columns and the liquid threads in their motion towards the heating surface. As a matter of fact, even the hydrodynamic process which takes place at the base of the vapour columns depends, through Q_o or δ_o on this interaction.

No further details based on the above calculation will be given for the present about the heat-transfer coefficient, for two reasons: (1) The process which takes place at the base of the vapour columns depends on the presence of active centres (the fact that the dry regions are not found only in the central zone but are also distributed over the whole base of the vapour column and that they appear and disappear, could be interpreted as an argument in favour of this assumption.* (2) It is not yet possible to compute Q_0, R', ϵ' and h_1 as functions of the hydrodynamic conditions. However, the calculation performed above gives certain indications concerning the mechanism by which surface forces influence boiling heat transfer and could be used eventually as a basis for obtaining dimensionless groups able to characterize boiling heat transfer at large fluxes.

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* In this sense it would be interesting to establish experimentally if the dry points appear always at the same spots.

† A synthesis paper of [3] and [7] has been published in *Int. J. Heat Mass Transfer* 7, 191 (1964).

Résumé—Dans la première partie de l'article, un modèle hydrodynamique est proposé pour le régime de bulles isolées. Le modèle qui est du type couche limite tient compte du mouvement circulatoire provoqué dans le liquide par le détachement des bulles. On obtient deux équations, l'une pour l'écoulement laminaire, l'autre pour l'écoulement turbulent.

Dans la deuxiéme partie est proposé un mé un mécanisme pour le transport de chaleur à des flux de chaleur élevés impliquant l'existencede colonnes de vapeur et le mouvement d'un film liquidem ince sous ces colonnes. Dans ce mécanisme, on considère que le flux de chaleur dèpend: (1) de l'intéraction hydrodynamique entre les filets liquides qui se dirigent vers la surface chauffante et les colonnes de vapeur; (2) du mouvement liquide en couche mince sous les colonnes de vapeur, sous l'action de forces superficielles; (3) de la vaporisation de ce film qui a lieu à l'interface liquide–vapeur et d'une façon explosive au voisinage des centres actifs.

Zusammenfassung—Im ersten Teil der Arbeit wird ein hydrodynamisches Modell vorgeschlagen für das Regime isolierter Blasen. Dieses Grenzschichtmodell berücksichtigt die Zirkulationsbewegungung in der Flüssigkeit die bei der Blasenablösung hervorgerufen wird. Zwei Gleichungen wurden gefunden, die eine gilt für laminare, die andere für turbulente Strömung.

Im zweiten Teil wird ein Mechanismus für den Wärmeübergang bei grossen Wärmestromdichten vorgeschlagen der die Existenz von Dampfsäulen und die Bewegung eines dünnen Flüssigkeitsfilms unter diesen Säulen einschliesst. Dabei soll der Wärmeübergang abhängen von: (1) der gegenseitigen hydrodynamischen Beeinflussung der Flüssigkeitsfäden die sich zur Heizfläche und den Dampfsäulen hinbewegen; (2) der Flüssigkeitsbewegung in einer dünnen Schicht unter den Dampfsäulen, hervorgerufen von Oberflächenkräften; (3) der Verdampfung dieses Films die an der Trennschicht zwischen Flüssigkeit und Dampf auftritt und explosionsartig in der Umgebung aktiver Zentren erfolgt.

Аннотация—В первой части статьи предлагается гидродинамическая модель режима изолированных пузырьков. Модель типа пограничного слоя учитывает циркуляционное движение жидкости, вызываемое отрывом пузырьков. Получены два уравнения, одно для ламинарного, а другое для турбулентного течения.

Во второй части предложен механизм переноса тепла при больших плотностях теплового потока, подразумевающий наличие колонок пузырьков пара и движение под ними тонкой пленки жидкости.

Предполагается, что при таком механизме скорость переноса тепла зависит от: (1) гидродинамического взаимодействия между нитями жидкости, движущимися к поверхности нагрева и колонками пузырьков пара: (2) движения тонкого слоя жидкости под колонками пузырьков пара под действием поверхностных сил: (3) испарения этой пленки, происходящего на границе раздела жидкость- пар и особенно вблизи активных центров.